C

1. Which of the following phenomena could be explained by classical physics and did *not* require a quantum hypothesis in order to make theory agree with experiment?

(a) Blackbody spectra
(b) Diffraction of light
(c) Low-temperature heat capacity in perfect crystals
(d) The photoelectric effect
(e) Gravity
(f) (b) and (e)
(g) (a), (b), and (d)
(h) None of the above

2. If a normalized wave packet Ψ is given as $\Psi(x, y, z, t) = \sum_{n=1}^{\infty} c_n \psi_n(x, y, z) e^{-iE_n t/\hbar}$, what is the probability that an experiment will cause the system to collapse to the specific stationary state j?

(a) Quantum mechanics does not allow you to know this probability
(b) One
(c) $\langle \psi_j | H | \psi_j \rangle$
(d) c_j^*
(e) $|c_j|^2$
(f) c_j
(g) (b) and (d)
(h) Only Schrödinger's cat knows

3. Which of the following statements about the de Broglie wavelength λ are *true*?

(a) $\lambda = h / p$
(b) λ decreases as mass increases if velocity is constant
(c) λ increases as momentum increases
(d) λ is a constant, like Planck's constant
(e) A particle that has zero velocity has an infinite de Broglie wavelength
(f) All of the above
(g) (a), (b) and (e)
(h) (c) and (d)

4. Which of the below equations can be *false* for an arbitrary pair of orthonormal functions f and g?

(a) $\langle |f|^2 \rangle \langle |g|^2 \rangle = 1$
(b) $\langle f | H | g \rangle = 0$
(c) $\langle f | g \rangle = 0$
(d) $fg = 0$
(e) $f^*g - g^*f = 0$
(f) (a) and (c)
(g) (b), (d) and (e)
(h) All of the above

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5. Which of the below expectation values are or may be non-zero?

(a)	$< \sin x \mid x \mid \cos x >$	(e)	$<f \mid g> - <g \mid f>^*$
(b)	$< \sin^2 x \mid x \mid \cos^2 x >$	(f)	$<\boldsymbol{\mu}_{mn}>$ for a forbidden transition
(c)	$< f \mid [A,B] \mid g >$ where A and B commute	(g)	(a), (d) and (e)
(d)	$< \Psi_m \mid H \mid \Psi_n >$ where Ψ_m and Ψ_n are non-degenerate stationary states	(h)	(b), (c), and (f)

6. Which of the following did Bohr assume in order to derive a model consistent with the photoemission spectra of one-electron atoms?

(a)	The electron is a delocalized wave	(e)	The angular momentum of the electron is quantized
(b)	The ionization potential is equal to the work function	(f)	(a) and (b)
(c)	The one-electron atom is like a particle in a box	(g)	(b) and (d)
(d)	The Coulomb potential is quantized	(h)	None of the above

7. Which of the following statements about a well behaved wave function is *true*?

(a)	It must be continuous	(e)	Its square modulus has units of probability density
(b)	It may take on complex values	(f)	It must be an eigenfunction of the momentum operator
(c)	It must be quadratically integrable	(g)	(d) and (f)
(d)	It must be equal to its complex conjugate	(h)	(a), (b), (c), and (e)

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8. Which of the following statements are *false* about the free particle?

(a)	Its Schrödinger equation is $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - E\right)\Psi(x) = 0$	(e)	Valid wave functions include $\Psi(x) = Ae^{ikx} + Be^{-ikx}$ where $k = \frac{\sqrt{2mE}}{\hbar}$
(b)	It may be regarded as having a wave function that is the superposition of a left-moving particle and a right-moving particle	(f)	Valid wave functions include $\Psi(x) = N\cos kx$ where N is a normalization constant and k is defined in (e) above
(c)	Its energy levels are all non-negative	(g)	All of the above
(d)	Its energy levels are quantized	(h)	None of the above

9. Given a particle of mass m in a box of length L having the wave function $\Psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, what is the energy of the level corresponding to n = 4?

(a)	Since this wave function is not an eigenfunction of the Hamiltonian the question cannot be answered	(e)	$8\hbar^2 / mL^2$
(b)	$< \Psi \mid p_x^2 \mid \Psi >$	(f)	(c) and (d)
(c)	16 times the energy of the ground state	(g)	(b) and (e)
(d)	$\frac{8\pi^2\hbar^2}{mL^2}$	(h)	None of the above

10. On which of the below functions does the parity operator Π act in the fashion $\Pi[f(x)] = (-1)f(x)$?

(a)	x	(e)	Any eigenfunction of the Hamiltonian
(b)	x^3	(f)	(b) and (d)
(c)	e^{ix}	(g)	(a), (b), and (d)
(d)	$\sin x$	(h)	(b), (d), and (e)

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Short-answer (20 points)

Prove that, given a pair of normalized but *not* orthogonal functions ψ_1 and ψ_2, the function $\psi_3 = \psi_2 - S\psi_1$ is orthogonal to ψ_1 if S is the overlap integral of ψ_1 and ψ_2. Is ψ_3 normalized? (Use the back of the page if necessary).

We evaluate

$$\begin{aligned} \langle\psi_1|\psi_3\rangle &= \langle\psi_1|\psi_2 - S\psi_1\rangle \\ &= \langle\psi_1|\psi_2\rangle - S\langle\psi_1|\psi_1\rangle \\ &= S - S \bullet 1 \\ &= 0 \end{aligned}$$

which proves the orthogonality (we use normalization of ψ_1 in going from line 2 to 3).

With respect to normalization, we evaluate

$$\begin{aligned} \langle\psi_3|\psi_3\rangle &= \langle\psi_2 - S\psi_1|\psi_2 - S\psi_1\rangle \\ &= \langle\psi_2|\psi_2\rangle - 2S\langle\psi_1|\psi_2\rangle + S^2\langle\psi_1|\psi_1\rangle \\ &= 1 - 2S \bullet S + S^2 \\ &= 1 - S^2 \end{aligned}$$

so ψ_3 is only normalized for the boring case of $S = 0$ (which means that ψ_1 and ψ_2 were orthogonal to begin with since no change to ψ_2 is required to generate ψ_3).

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