1. Which of the following phenomena could be explained by classical physics and did not require a quantum hypothesis in order to make theory agree with experiment?

(a) Blackbody spectra
(b) Diffraction of light
(c) Low-temperature heat capacity in perfect crystals
(d) The photoelectric effect
(e) Atomic line spectra
(f) (b) and (e)
(g) (a), (b), and (d)
(h) (c) and (d)

2. If a normalized wave packet $\Psi$ is given as $\Psi(x, y, z, t) = \sum_{n=1}^{\infty} c_n \psi_n(x, y, z)e^{-iE_n t / h},$ what is the probability that an experiment will cause the system to collapse to the specific stationary state $j$?

(a) Quantum mechanics does not allow you to know this probability
(b) One
(c) $<\psi_j | H | \psi_j>$
(d) $c_j^*$
(e) $c_j$
(f) $|c_j|^2$
(g) (b) and (d)
(h) Only Schrödinger’s cat knows

3. Which of the following statements about the de Broglie wavelength $\lambda$ are false?

(a) $\lambda$ decreases as mass increases if velocity is constant
(b) $\lambda = h / p$
(c) $\lambda$ increases as momentum increases
(d) $\lambda$ is a constant, like Planck’s constant
(e) A particle that has zero velocity has an infinite de Broglie wavelength
(f) All of the above
(g) (a), (b) and (e)
(h) (c) and (d)

4. Which of the following statements about a well behaved wave function is true?

(a) It must be continuous
(b) It may take on complex values
(c) It must be quadratically integrable
(d) It must be equal to its complex conjugate
(e) Its square modulus has units of probability density
(f) It must be an eigenfunction of the momentum operator
(g) (d) and (f)
(h) (a), (b), (c), and (e)
5. Which of the below equations will be true for any pair of orthonormal functions $f$ and $g$ that are eigenfunctions of the Hamiltonian $H$?

(a) $fg = 0$  
(b) $< |f|^2 > < |g|^2 > = 1$  
(c) $f^*g - g^*f = 0$  
(d) $< f | H | g > = 0$  
(e) $< f | g > = 0$  
(f) (a) and (c)  
(g) (b), (d) and (e)  
(h) All of the above

6. Which of the below expectation values are or may be non-zero?

(a) $< \sin x | x | \cos x >$  
(b) $< \sin^2 x | x | \cos^2 x >$  
(c) $< f | [A,B] | g >$ where $A$ and $B$ commute  
(d) $< \Psi | H | \Psi >$ where $\Psi$ is a stationary state  
(e) $< f | g > - < g | f >$  
(f) $< \mu_{mn} >$ for a forbidden transition  
(g) (a), (d) and (e)  
(h) (b), (c), and (f)

7. Which of the following did Bohr assume in order to derive a model consistent with the photoemission spectra of one-electron atoms?

(a) The electron is a delocalized wave  
(b) The angular momentum of the electron is quantized  
(c) The one-electron atom is like a particle in a box  
(d) The Coulomb potential is quantized  
(e) The ionization potential is equal to the work function  
(f) (a) and (b)  
(g) (b) and (d)  
(h) None of the above
8. Which of the following statements are *false* about the free particle?

(a) Its Schrödinger equation is \( \left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - E \right) \Psi(x) = 0 \)

(b) It may be regarded as having a wave function that is the superposition of a left-moving particle and a right-moving particle.

(c) Its energy levels are all non-negative.

(d) Its energy levels are not quantized.

(e) Valid wave functions include \( \Psi(x) = Ae^{ikx} + Be^{-ikx} \) where \( k = \frac{\sqrt{2mE}}{\hbar} \)

(f) Valid wave functions include \( \Psi(x) = N \cos kx \) where \( N \) is a normalization constant and \( k \) is defined in (e) above.

(g) All of the above.

(h) None of the above.

9. Given a particle of mass \( m \) in a box of length \( L \) having the wave function \( \Psi(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \), what is the energy of the level corresponding to \( n = 4 \)?

(a) Since this wave function is not an eigenfunction of the Hamiltonian, the question cannot be answered.

(b) \( 16 \) times the energy of the ground state.

(c) \( \langle \Psi | p_x^2 | \Psi \rangle \)

(d) \( 8\hbar^2 / mL^2 \)

(e) \( \frac{8\pi^2 \hbar^2}{mL^2} \)

(f) (c) and (d).

(g) (b) and (e).

(h) None of the above.

10. On which of the below functions does the parity operator \( \Pi \) act in the fashion \( \Pi[f(x)] = (-1)f(x) \)?

(a) \( x \)

(b) \( x^2 \)

(c) \( e^{ix} \)

(d) \( \cos x \)

(e) Any eigenfunction of the Hamiltonian.

(f) (b) and (d).

(g) (a), (b), and (d).

(h) (b), (d), and (e).

NAME: ________________________________________________________________
Short-answer (20 points)

Prove that, given a pair of normalized but *not* orthogonal functions $\psi_1$ and $\psi_2$, the function $\psi_3 = \psi_2 - S\psi_1$ is orthogonal to $\psi_1$ if $S$ is the overlap integral of $\psi_1$ and $\psi_2$. Is $\psi_3$ normalized? (Use the back of the page if necessary).

We evaluate

\[
\langle \psi_1 | \psi_3 \rangle = \langle \psi_1 | \psi_2 - S\psi_1 \rangle \\
= \langle \psi_1 | \psi_2 \rangle - S \langle \psi_1 | \psi_1 \rangle \\
= S - S \cdot 1 \\
= 0
\]

which proves the orthogonality (we use normalization of $\psi_1$ in going from line 2 to 3).

With respect to normalization, we evaluate

\[
\langle \psi_3 | \psi_3 \rangle = \langle \psi_2 - S\psi_1 | \psi_2 - S\psi_1 \rangle \\
= \langle \psi_2 | \psi_2 \rangle - 2S \langle \psi_1 | \psi_2 \rangle + S^2 \langle \psi_1 | \psi_1 \rangle \\
= 1 - 2S \cdot S + S^2 \\
= 1 - S^2
\]

so $\psi_3$ is only normalized for the boring case of $S = 0$ (which means that $\psi_1$ and $\psi_2$ were orthogonal to begin with since no change to $\psi_2$ is required to generate $\psi_3$).