

**Chemistry 3502/4502**

**Exam II Key**

**October 10, 2003**

- 1) This is a multiple choice exam. Circle the correct answer.**
- 2) There is *one* correct answer to every problem. There is no partial credit.**
- 3) A table of useful integrals and other formulae is provided at the end of the exam.**
- 4) You should try to go through all the problems first, saving harder ones for later.**
- 5) There are 20 problems. Each is worth 5 points.**
- 6) There is no penalty for guessing.**
- 7) Please write your name at the bottom of each page.**
- 8) Please mark your exam with a pen, not a pencil. Do not use correction fluid to change an answer. Cross your old answer out and circle the correct answer. Exams marked with pencil or correction fluid will not be eligible for regrade under any circumstances.**

**Score on Next Page after Grading**

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1. How many nodes does the QMHO wave function

$$\Psi(x) = \left( \frac{\sqrt{k\mu}}{147456\hbar\pi} \right)^{1/4} \left[ 16 \left( \frac{\sqrt{k\mu}}{\hbar} \right)^2 x^4 - 48 \left( \frac{\sqrt{k\mu}}{\hbar} \right) x^2 + 12 \right] e^{-\sqrt{k\mu}x^2/2\hbar}$$

- |       |                         |
|-------|-------------------------|
| (a) 0 | (e) 4                   |
| (b) 1 | (f) It depends on $k$   |
| (c) 2 | (g) It depends on $\mu$ |
| (d) 3 | (h) (f) and (g)         |

2. For the diatomic molecule BeO, where Be has atomic mass 9 and O has atomic mass 16, what is the reduced mass?

- |                                |  |
|--------------------------------|--|
| (a) $(9 \times 16) / (9 + 16)$ | (e) 1                                    |
| (b) 9                          | (f) It depends on $k$                    |
| (c) 16                         | (g) It depends on the vibrational energy |
| (d) $(9 + 16) / (9 \times 16)$ | (h) None of the above                    |

3. Which of the following statements is/are true about  $\langle x^2 \rangle$  evaluated for one-dimensional QMHO wave functions over the same potential  $V$ ?

- |   |  |
|---|--|
| (a) Parity requires it to be zero for levels where $n$ is odd | (e) It is equal to $2\langle V \rangle/k$ where $k$ is the oscillator force constant |
| (b) It increases with increasing $n$                          | (f) (b) and (c)  |
| (c) It is equal to $\langle x \rangle^2$                      | (g) (c) and (d)  |
| (d) It is always positive                                     | (h) (b), (d), and (e)  |

4. Which of the following statements about chemically equivalent CH vs. CD bonds is/are true?

- |   |   |
|---|---|
| (a) The CD bond has the smaller zero-point vibrational energy       | (e) The CH bond has the larger reduced mass |
| (b) The CH bond has the smaller vibrational frequency               | (f) All of the above                        |
| (c) The CH bond has the smaller force constant $k$ by a factor of 2 | (g) (a), (b) and (e)                        |
| (d) Both bonds absorb photons at the same IR frequency              | (h) (c) and (d)                             |

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5. Which of the following statements is/are true for a given set of QMHO wave functions corresponding to the same harmonic potential  $V$ ?
- |  |   |
|--|---|
| (a) The ground state energy is above the bottom of the potential               | (e) The wave functions are eigenfunctions of the parity operator                |
| (b) The number of nodes is equal to $n+1$ , where $n$ is the energy level      | (f) The selection rule for spectroscopic transitions is $n \rightarrow n \pm 1$ |
| (c) $\langle T \rangle_n = \langle V \rangle_n = (1/2)\langle E \rangle_n$     | (g) (a), (c), (e), and (f)  |
| (d) The wave functions have zero amplitude beyond the classical turning points | (h) All of the above  |
6. What is  $[L_x, L_y]$  when  $|\mathbf{L}|^2 > 0$ ?
- |                         |   |
|-------------------------|---|
| (a) 0                   | (e) $i\hbar(L^2 - L_x^2 - L_y^2)^{1/2}$ |
| (b) $i\hbar$            | (f) 1                                   |
| (c) $i\hbar L_z$        | (g) (c) and (e)                         |
| (d) $i\hbar(L_x - L_y)$ | (h) None of the above                   |
7. Which of the following statements about angular momentum operators, eigenvalues, and eigenfunctions is/are *false*?
- |   |                       |
|---|-----------------------|
| (a) $L_+ = (L_-)^*$   | (e) $L_- Y_{l,l} = 0$ |
| (b) $\langle L^2 \rangle = \langle L_z \rangle^2$ if $m_l = l$        | (f) (b) and (e)       |
| (c) For each value of $l$ there are $2l + 1$ possible values of $m_l$ | (g) (b), (d) and (e)  |
| (d) $L_+ Y_{l,l} = 0$   | (h) None of the above |
8. Which of the following statements about the spherical harmonics  $Y$  is/are *true*?
- |   |   |
|---|---|
| (a) The number of nodes in $Y_{l,m_l}$ is $l$   | (e) $Y_{0,0}$ is a constant everywhere in space |
| (b) $\langle Y_{l,0}   T   Y_{l,0} \rangle > \langle Y_{l',0}   T   Y_{l',0} \rangle$ if $l > l'$ | (f) (a) and (e)                                 |
| (c) The <i>real</i> spherical harmonics are not all eigenfunctions of $L_z$                       | (g) (a), (b), (c) and (e)                       |
| (d) The <i>complex</i> spherical harmonics are not all eigenfunctions of $L^2$                    | (h) All of the above                            |

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9. What is the eigenvalue of  $(L_x^2 + L_y^2)$  for  $\Psi$  if the eigenvalue of  $L^2$  for  $\Psi$  is  $12\hbar^2$  and the eigenvalue of  $L_z$  for  $\Psi$  is  $2\hbar$ ?

- |  |                       |
|--|-----------------------|
| (a) The Heisenberg uncertainty principle dictates that $\Psi$ cannot be an eigenfunction for $(L_x^2 + L_y^2)$ | (e) $10\hbar$         |
| (b) $10\hbar^2$  | (f) 0                 |
| <span style="border: 1px solid black; padding: 2px;">(c) <math>8\hbar^2</math></span>                          | (g) $\pi$             |
| (d) $4\hbar$   | (h) None of the above |

10. The frequency for the ground- to first-excited-state rotational transition in HO ( $m_H = 1$ ;  $m_O = 16$ ) is  $100 \text{ cm}^{-1}$ . What is the frequency for the same transition in DS ( $m_D = 2$ ;  $m_S = 32$ ) if HO and DS have exactly the same bond length?

- |   |   |
|---|---|
| (a) $400 \text{ cm}^{-1}$   | (e) $25 \text{ cm}^{-1}$                                  |
| (b) $200 \text{ cm}^{-1}$   | (f) It depends on the bond length                         |
| (c) $100 \text{ cm}^{-1}$   | (g) It cannot be determined from the information provided |
| <span style="border: 1px solid black; padding: 2px;">(d) <math>50 \text{ cm}^{-1}</math></span> | (h) (f) and (g)   |

11. For a diatomic molecule, what is the rotational constant  $B$ ?

- |                             |  |
|-----------------------------|--|
| (a) The eigenvalue of $L^2$ | <span style="border: 1px solid black; padding: 2px;">(e) <math>\hbar^2 / 2\mu R^2</math> where <math>\mu</math> is the reduced mass and <math>R</math> is the bond length</span> |
| (b) The eigenvalue of $L_z$ | (f) $\hbar^2 / I$ where $I$ is the moment of inertia   |
| (c) 0                       | (g) (e) and (f)  |
| (d) $J + K$                 | (h) None of the above  |

12. What are the diffuse interstellar bands?

- |   |   |
|---|---|
| (a) Groups of widely separated wandering alien musicians  | (e) Nebulae connecting one star to another                      |
| <span style="border: 1px solid black; padding: 2px;">(b) Absorption or emission peaks in the otherwise uniform cosmic spectrum</span> | (f) Regions of anomalous space-time                             |
| (c) Gas clouds of CO  | (g) Cosmic radio stations that are very difficult to tune in to |
| (d) The stripes on Schrödinger's cat  | (h) None of the above   |

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13. For a spin-free hydrogenic wave function, which of the below relationships between quantum numbers is/are always true?

- |                      |                       |
|----------------------|-----------------------|
| (a) $n = l > m_l$    | (e) $n = l + m_l$     |
| (b) $n > l > m_l$    | (f) (b) and (c)       |
| (c) $n > l + m_l$    | (g) (b) and (e)       |
| (d) $n > l \geq m_l$ | (h) None of the above |

14. What is the ground-state ionization potential for a one-electron atom having atomic number  $Z$ ?

- |  |   |
|--|---|
| (a) $Z^2$ a.u.   | (e) 1 a.u.  |
| (b) The negative of the energy of the electron in the 1s orbital | (f) The energy required to infinitely separate the nucleus and electron |
| (c) $(1/2)Z^2$ a.u.  | (g) (b), and (d)  |
| (d) $2Z^2$ a.u.  | (h) (b), (c), and (f)   |

15. Which of the following is not equal to 1.0 in atomic units?

- |                                |   |
|--------------------------------|---|
| (a) $\hbar$                    | (e) $0.529 \text{ \AA}$                                       |
| (b) $4\pi\epsilon_0$           | (f) The speed of light  |
| (c) The charge of the electron | (g) Twice the ionization potential of the ground-state H atom |
| (d) The mass of the electron   | (h) None of the above   |

16. Which of the below statements about electron spin is/are true?

- |  |   |
|--|---|
| (a) Spin is a necessary property if relativity is included in the electronic Schrödinger equation  | (e) Spin-orbit coupling is proportional to the 4th power of the atomic number |
| (b) Spin couples with orbital angular momentum according to $\mathbf{J} = \mathbf{L} + \mathbf{S}$ | (f) (a) and (c)   |
| (c) For a single electron, the only eigenvalues of $S_z$ are $\pm(1/2)\hbar$                       | (g) (a), (c), and (d)   |
| (d) Stern and Gerlach discovered electron spin by studying the magnetic moments of Ag atoms        | (h) All of the above  |

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17. An electron of spin  $\beta$  is in a 4f orbital. Which of the below sets of quantum numbers ( $n, l, m_l, m_s$ ) might describe such an electron?

- |     |                 |     |                       |
|-----|-----------------|-----|-----------------------|
| (a) | (4, 4, 4, 4)    | (e) | (4, 4, 3, -1/2)       |
| (b) | (4, 3, 2, 1/2)  | (f) | (c) and (e)           |
| (c) | (4, 3, 0, -1/2) | (g) | (b), (c), (d) and (e) |
| (d) | (4, 3, 0, 7/2)  | (h) | None of the above     |

18. Which of the following statements is/are true for a one-electron atom placed in a uniform magnetic field in the  $z$  direction?

- |     |   |     |  |
|-----|---|-----|--|
| (a) | Given a strong enough magnetic field, a $3^2P_{3/2}$ state can be lower in energy than a $3^2P_{1/2}$ state     | (e) | The proper Schrödinger equation includes the magnetic field strength |
| (b) | The magnetic field lifts the degeneracy of levels otherwise differing only in $m_j$                             | (f) | (b), (c), and (d)  |
| (c) | The phenomenon in answer (b) is called the Zeeman effect  | (g) | (b), (c), (d), and (e)   |
| (d) | New spectroscopic transitions will become available according to the selection rule $\Delta m_j = 0$ or $\pm 1$ | (h) | All of the above   |

19. The Landé  $g$  factor is  $g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$ . What  $g$  factors are possible for an  $\alpha$  electron in a 3p orbital?

- |     |           |     |  |
|-----|-----------|-----|--|
| (a) | 1         | (e) | 2/3 and 4/3  |
| (b) | 2         | (f) | It depends on the magnetic field strength                            |
| (c) | 5/3       | (g) | Insufficient information is supplied in order to answer the question |
| (d) | 2 and 5/2 | (h) | An infinite number of $g$ factors                                    |

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20. Which of the following wave functions has the greatest degeneracy?

- |  |   |
|--|---|
| (a) Particle in a box, level $n = 8$                       | (e) Spin-free hydrogenic wave function, $n = 3$ |
| (b) Quantum mechanical harmonic oscillator, level $n = 25$ | (f) Relativistic free electron at rest          |
| (c) Rigid rotator, $l = 5$                                 | (g) All of the above are singly degenerate      |
| (d) Spin-free hydrogenic wave function, $n = 6, l = 1$     | (h) (deliberately left blank)                   |

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